Quantum Nonlocality - a Mystery and its Applications

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Announcement of the 2022 Nobel Prize in Physics

KUNGL. VETENSKAPS-AKADEMIEN

THE ROYAL SWEDISH ACADEMY OF SCIENCES

4 OCTOBER 2022

Scientific Background on the Nobel Prize in Physics 2022

"FOR EXPERIMENTS WITH ENTANGLED PHOTONS, ESTABLISHING THE VIOLATION OF BELL INEQUALITIES AND PIONEERING QUANTUM INFORMATION SCIENCE"

The Nobel Committee for Physics

One of 34 co-authors of one of 41 cited papers...

35. B. Hensen, H. Bernien, A.E. Dréau, A. Reiserer, N. Kalb, M.S Blok, J. Ruitenberg, R.F.L Vermeulen, R.N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M.W. Mitchell, M. Markham D.J. Twitchen, D. Elkouss, S. Wehner, T.H. Taminiau and R. Hanson, *Nature* **526**, 682 (2015).

36. M. Giustina, M. A. M. Versteegh, S. Wengerowsky, J. Handsteiner, A. Hochrainer, K. Phelan, F. Steinlechner, J. Kofler, J.-Å. Larsson, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, J. Beyer, T. Gerrits, A. E. Lita, L.K. Shalm, S. W. Nam, T. Scheidl, R. Ursin, B. Wittmann and A. Zeilinger *Phys. Rev. Lett.* **115**, 250401 (2015).

37. L.K. Shalm, E. Meyer-Scott, B.G. Christensen, P. Bierhorst, M.A. Wayne, M.J. Stevens, T. Gerrits, S. Glancy, D.R. Hamel, M.S. Allman, K.J. Coakley, S.D. Dyer, C. Hodge, A.E. Lita, V.B. Verma, C. Lambrocco, E. Tortorici, A.L. Migdall, Y. Xhang, D.R. Kumor, W.H. Farr, F. Marsili, M.D. Shaw, J.A. Stern, C. Abellán, W. Amaya, V. Pruneri, T. Jennewein, M.W. Mitchell, PG. Kwiat, J.C. Bienfang, R.P. Mirin, E. Knill and S. W. Nam, *Phys. Rev. Lett.* **115**, 250402 (2015).

38. W. Rosenfeld, D. Burchardt, R. Garthoff, K. Redeker, N. Ortegel, M. Rau and H. Weinfurter, *Phys. Rev. Lett.* **119**, 010402 (2017).

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17 (18)

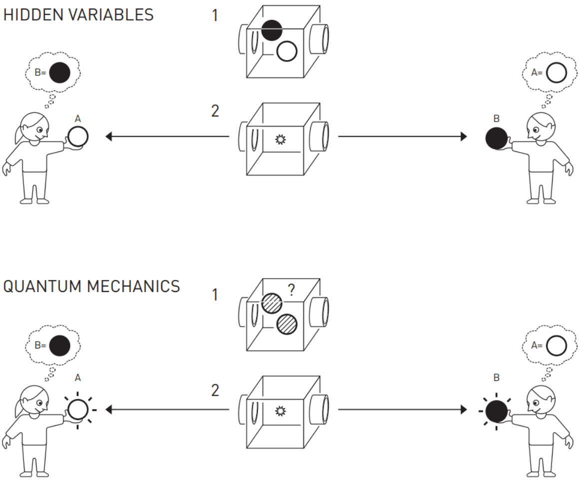
$$\frac{1}{34} \times \frac{1}{41} = \frac{1}{1394}$$

What was the Nobel Prize awarded for?

Talk outline:

- The Phenomenon: Bell nonlocality
- Nobel Laureate Work on Experimental Tests
- Current Research: Applications, Quantum Networks

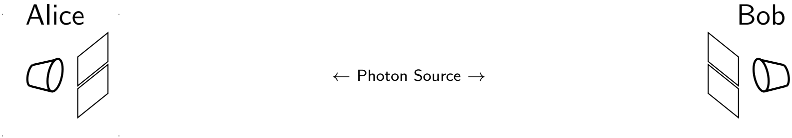
Bell Nonlocality



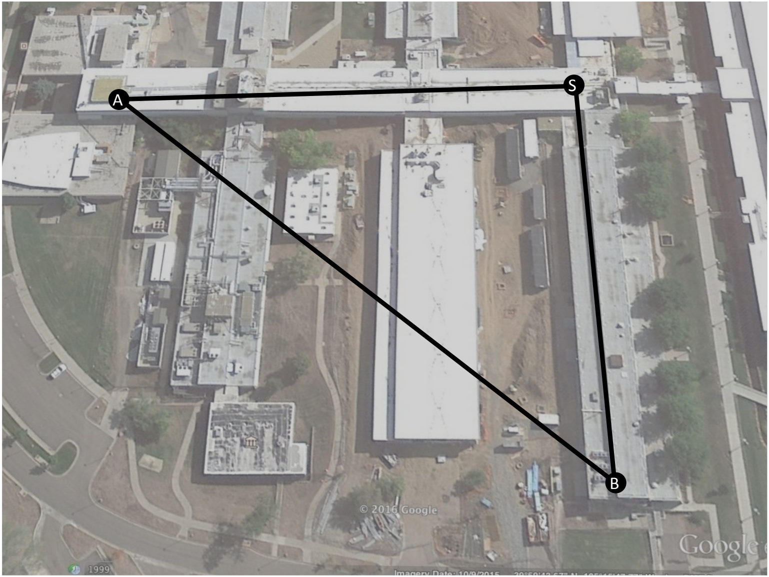
Bell Nonlocality

- Einstein-Podolsky-Rosen (1935): This is stupid. It is obviously the top way
- John Bell (1964): For a certain experiment with multiple different measurements, it *can't* be the top way
- Nobel Laureates: Clauser, Aspect, and Zeilinger do experiments with these measurements in 70s, 80s, 90s; final definitive tests in 2015

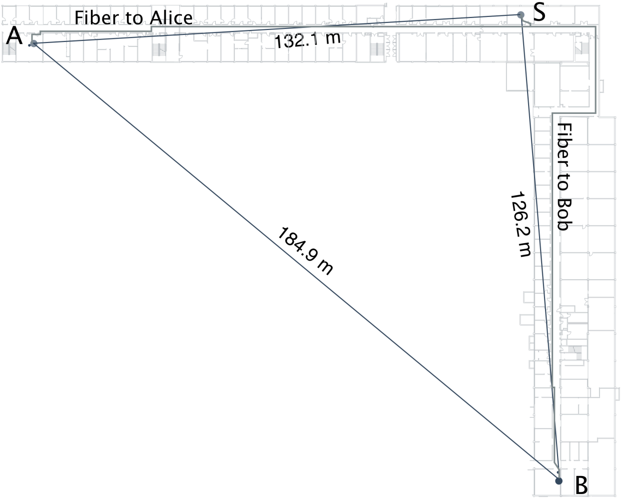
Bell Nonlocality



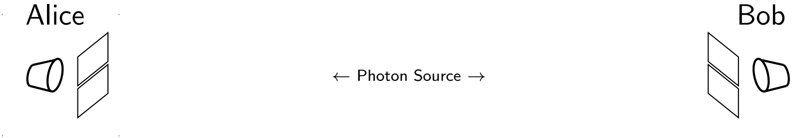
Bell Nonlocality



Bell Nonlocality



Bell Nonlocality

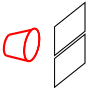


Bell Nonlocality

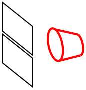
Both detectors click ("++")

Bell Nonlocality

Alice



Bob



Both detectors click ("++")

Outcomes:

++

Bell Nonlocality

Alice-only click (“+0”)

Outcomes:

++   +0

Bell Nonlocality

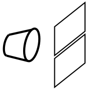
Bob-only click ("0+")

Outcomes:

++ +0 0+

Bell Nonlocality

Alice



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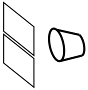


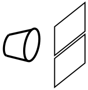
Neither detector clicks (“00”)

Outcomes:

$++ \quad +0 \quad 0+ \quad 00$

Bell Nonlocality

Alice



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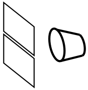


Alice measures Q

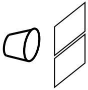
Bob measures S

Outcomes:

$++ \quad +0 \quad 0+ \quad 00$

Bell Nonlocality

Alice



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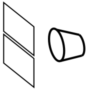


Alice measures Q

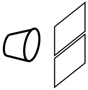
Bob measures S

Outcomes:

	$++$	$+0$	$0+$	00
setting$=QS$				

Bell Nonlocality

Alice



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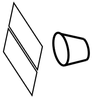


Alice measures Q

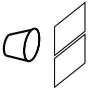
Bob measures T

Outcomes:

	$++$	$+0$	$0+$	00
setting$=QS$				

Bell Nonlocality

Alice



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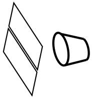


Alice measures Q

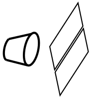
Bob measures T

Outcomes:

	$++$	$+0$	$0+$	00
setting=QS				
QT				

Bell Nonlocality

Alice



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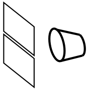


Alice measures R

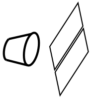
Bob measures S

Outcomes:

	$++$	$+0$	$0+$	00
setting=QS				
QT				

Bell Nonlocality

Alice



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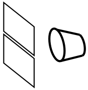


Alice measures R

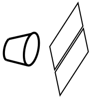
Bob measures S

Outcomes:

	$++$	$+0$	$0+$	00
setting$=QS$				
QT				
RS				

Bell Nonlocality

Alice



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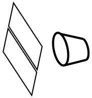

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Outcomes:

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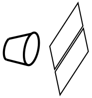
setting$=QS$				
QT				
RS				

Bell Nonlocality

Alice



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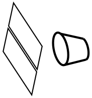


Alice measures R

Bob measures T

Outcomes:

	$++$	$+0$	$0+$	00
setting$=QS$				
QT				
RS				
RT				

Bell Nonlocality

A simple example (each row is a probability distribution)

	$++$	$+0$	$0+$	00
QS	1	0	0	0
QT	1	0	0	0
RS	1	0	0	0
RT	1	0	0	0

Bell Nonlocality

Another simple example:

	$++$	$+0$	$0+$	00
QS	0	0	0	1
QT	0	0	0	1
RS	0	0	0	1
RT	0	0	0	1

Bell Nonlocality

Slightly more-complicated examples:

	$++$	$+0$	$0+$	00
QS	1	0	0	0
QT	1	0	0	0
RS	0	0	1	0
RT	0	0	1	0

Bell Nonlocality

Slightly more-complicated examples:

	$++$	$+0$	$0+$	00
QS	$1/2$	0	0	$1/2$
QT	$1/2$	0	0	$1/2$
RS	$1/2$	0	0	$1/2$
RT	$1/2$	0	0	$1/2$

Bell Nonlocality

In a *local hidden variable theory*, the outcomes are functions of pre-existing variables λ_i

	Q	R	S	T
λ_1	$+$	$+$	$+$	$+$
λ_2	$+$	$+$	$+$	0
λ_3	$+$	$+$	0	$+$
λ_4	$+$	$+$	0	0
λ_5	$+$	0	$+$	$+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
λ_{16}	0	0	0	0

Bell Nonlocality

In a *local hidden variable theory*, the outcomes are functions of pre-existing variables λ_i

	Q	R	S	T
λ_1	$+$	$+$	$+$	$+$
λ_2	$+$	$+$	$+$	0
λ_3	$+$	$+$	0	$+$
λ_4	$+$	$+$	0	0
λ_5	$+$	0	$+$	$+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
λ_{16}	0	0	0	0

	$++$	$+0$	$0+$	00
QS	1	0	0	0
QT	1	0	0	0
RS	1	0	0	0
RT	1	0	0	0

Bell Nonlocality

In a *local hidden variable theory*, the outcomes are functions of pre-existing variables λ_i

	Q	R	S	T
λ_1	$+$	$+$	$+$	$+$
λ_2	$+$	$+$	$+$	0
λ_3	$+$	$+$	0	$+$
λ_4	$+$	$+$	0	0
λ_5	$+$	0	$+$	$+$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
λ_{16}	0	0	0	0

	$++$	$+0$	$0+$	00
QS	1	0	0	0
QT	1	0	0	0
RS	0	0	1	0
RT	0	0	1	0

Bell Nonlocality

In a *local hidden variable theory*, the outcomes are functions of pre-existing variables λ_i

	Q	R	S	T
λ_1	+	+	+	+
λ_2	+	+	+	0
λ_3	+	+	0	+
λ_4	+	+	0	0
λ_5	+	0	+	+
⋮	⋮	⋮	⋮	⋮
λ_{16}	0	0	0	0

	++	+0	0+	00
QS	1/2	0	0	1/2
QT	1/2	0	0	1/2
RS	1/2	0	0	1/2
RT	1/2	0	0	1/2

The Clauser-Horne-Shimony-Holt Inequality

No local hidden variable can yield more than three "1" entries in the X regions:

	$++$	$+0$	$0+$	00
QS	X			X
QT	X			X
RS	X			X
RT		X	X	

Clauser-Horne-Shimony-Holt Inequality:
$P(A = B|QS)+P(A = B|QT)+P(A = B|RS)+P(A \neq B|RT) \leq 3$
Quantum Value: 3.416

John Clauser

- 1969 CHSH
- 1972 Freedman-Clauser Experiment
- 1974 Clauser-Horne paper

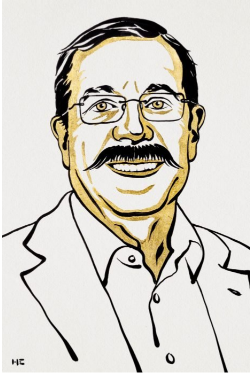
John Clauser

The 1972 Experiment:

- First Bell experiment
- Uses entangled photons
- Issues: fixed settings, low detection

Alain Aspect

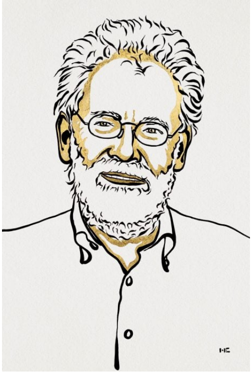
- 1982 Bell experiment
- Uses photons
- Fast settings changes (10 ns)



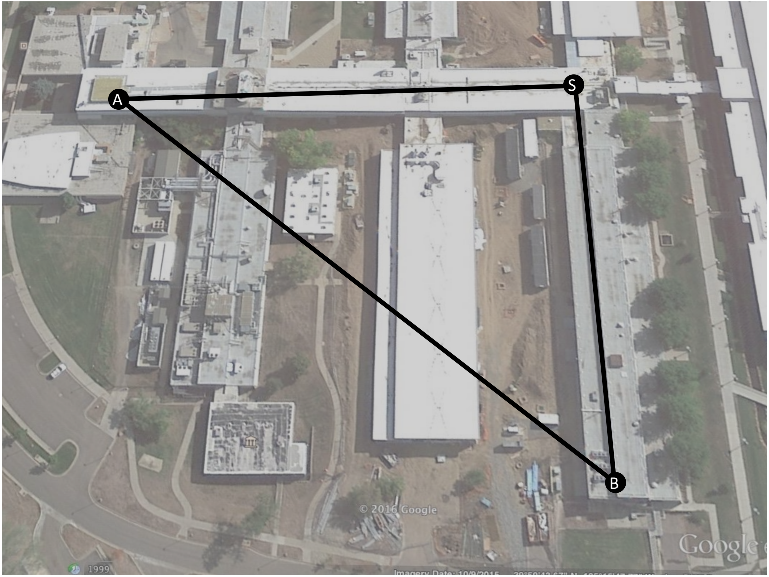
Anton Zeilinger

1998 Bell experiment with random setting changes. Also, applications:

- 1997 Quantum Teleportation experiment; repeaters
- 2016-2017 Satellite entanglement
- 1989 Greenberger, Horne, Zeilinger multi-party entanglement
- 1999, 2000 GHZ Experiments
- 2006 QKD Experiment



My work: 2015 Experiment



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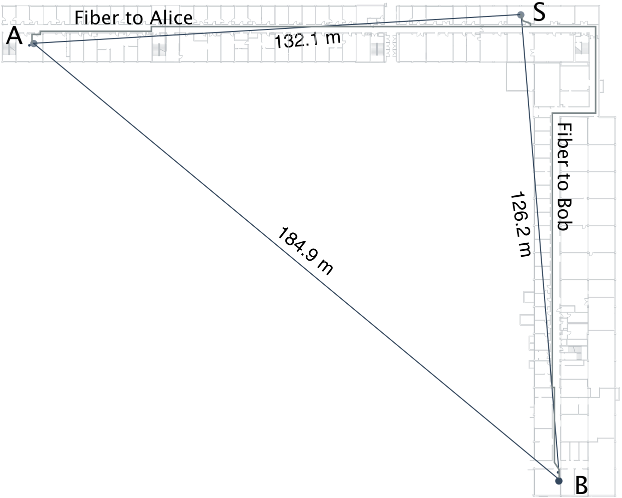

Dealing with Noisy Data

CH-Eberhard

	++	+0	0+	00
QS	+			
QT		-		
RS			-	
RT	-			

Outcomes

Settings	++	+0	0+	00
ab	6378	3289	3147	44336240
ab'	6794	2825	23230	44311018
$a'b$	6486	21358	2818	44302570
$a'b'$	106	27562	30000	44274530

CH-E Inequality:

$$P(++|QS) - P(+0|QT) - P(0+|RS) - P(00|RT) \leq 0$$

Quantum can exceed 0

My Work

CH-E Inequality:

$$P(++|QS) - P(+0|QT) - P(0+|RS) - P(00|RT) \leq 0$$

- Memory Robustness (CHSH[1] and CH-E[2])
- "Ignoring" unmentioned CH-E trials is OK (direct proof,[2] Doob's stopping theorem)

[1] P. Bierhorst, "A Rigorous Analysis of the Clauser–Horne–Shimony–Holt Inequality Experiment When Trials Need Not be Independent" Foundations of Physics 44 (7):736-761 (2014)
[2] P. Bierhorst, "A robust mathematical model for a loophole-free Clauser–Horne experiment" J. Phys. A: Math. Theor. 48: 195302 (2015)

Loophole Free Bell experiments

35. B. Hensen, H. Bernien, A.E. Dréau, A. Reiserer, N. Kalb, M.S Blok, J. Ruitenberg, R.F.L Vermeulen, R.N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M.W. Mitchell, M. Markham D.J. Twitchen, D. Elkouss, S. Wehner, T.H. Taminiau and R. Hanson, *Nature* **526**, 682 (2015).

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What does it all mean?

What does it all mean?

“True” randomness in nature, appearing nonlocally

Randomness Intuition

Now consider the following special distribution:

	$++$	$+0$	$0+$	00
ab	$1/2$	0	0	$1/2$
ab'	$1/2$	0	0	$1/2$
$a'b$	$1/2$	0	0	$1/2$
$a'b'$	0	$1/2$	$1/2$	0

Randomness Intuition

A possible decomposition?

	$++$	$+0$	$0+$	00
ab	1	0	0	0
ab'	1	0	0	0
$a'b$	1	0	0	0
$a'b'$	0	1	0	0

and

	$++$	$+0$	$0+$	00
ab	0	0	0	1
ab'	0	0	0	1
$a'b$	0	0	0	1
$a'b'$	0	0	1	0

Randomness Intuition

A possible decomposition?

	$++$	$+0$	$0+$	00
ab	1	0	0	0
ab'	1	0	0	0
$a'b$	1	0	0	0
$a'b'$	0	1	0	0

and

	$++$	$+0$	$0+$	00
ab	0	0	0	1
ab'	0	0	0	1
$a'b$	0	0	0	1
$a'b'$	0	0	1	0

When Bob chooses b', Alice can signal Bob.

Randomness Intuition

But if we don't try to decompose the distribution, Bob's probability of "+" is independent of Alice's setting choice.

	$++$	$+0$	$0+$	00
ab	$1/2$	0	0	$1/2$
ab'	$1/2$	0	0	$1/2$
$a'b$	$1/2$	0	0	$1/2$
$a'b'$	0	$1/2$	$1/2$	0

Randomness Intuition

But if we don't try to decompose the distribution, Bob's probability of "+" is independent of Alice's setting choice.

	$++$	$+0$	$0+$	00
ab	$1/2$	0	0	$1/2$
ab'	$1/2$	0	0	$1/2$
$a'b$	$1/2$	0	0	$1/2$
$a'b'$	0	$1/2$	$1/2$	0

If we disallow signaling between Alice and Bob, we must accept that there is randomness in the distribution.

Ongoing Research

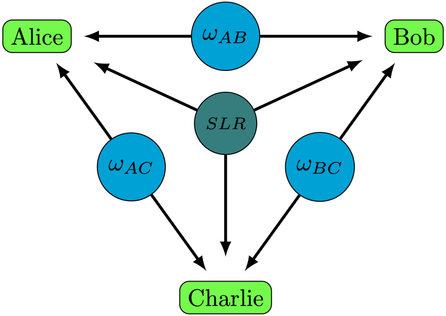
- Randomness generation using a loophole-free Bell test[1]
- Device-independent quantum secured communication
- New Measures of Quantum Nonlocality for Multiple Parties[2]
- Using Genuine Multi-Party Nonlocality in Quantum Networks[3]

[1] Experimentally generated randomness certified by the impossibility of superluminal signals, Bierhorst et al., Nature 556:223 (2018)

[2] Ruling out bipartite nonsignaling nonlocal models for tripartite correlations, Bierhorst, Phys. Rev. A 104:012210 (2021)

[3] Hierarchy of Multipartite Nonlocality and Device-Independent Effect Witnesses, Bierhorst and Prakash, Phys. Rev. Lett. 130:250201 (2023)

Quantum Networks and Genuine Multipartite Nonlocality



- GMNL
- Entangled Measurements
- Applications?

Thank You

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