Ramanujan's Notebooks

Part I

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Bruce C. Berndt

Ramanujan's Notebooks

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Ramanujan's Lost Notebook

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EDITED BY

G. H. HARDY,

P. V. SESHU AIYAR

& B. M. WILSON

Collected Papers of Srinivasa Ramanujan

If $\phi = \frac{x}{1+}\,\frac{x^5}{1+}\,\frac{x^{10}}{1+}$ &c and $f = \frac{x^{\frac{1}{5}}}{1+}\,\frac{x}{1+}\,\frac{x^2}{1+}\,\frac{x^3}{1+}$ &c

then $f^5 = \phi \cdot \frac{1-2\phi+4\phi^2-3\phi^3+\phi^4}{1+3\phi+4\phi^2+2\phi^3+\phi^4}$.

$\sqrt{505}$ $\quad (\sqrt{5}-2)^{12}(\sqrt{101}-10)^6\left(\frac{5\sqrt{5}+\sqrt{101}}{4} \pm \sqrt{\frac{105+5\sqrt{505}}{8}}\right)^{12}$

$\sqrt{38}$ $\quad \frac{g^3+g\sqrt{2}}{1+g^2\sqrt{2}} = \sqrt{1+\sqrt{2}}$

$\sqrt{26}$ $\quad \frac{g^3+g\cdot\frac{\sqrt{13}+1}{2}}{1+g^2\cdot\frac{\sqrt{13}-1}{2}} = \sqrt{\frac{3+\sqrt{13}}{2}}$

$\sqrt{50}$ $\quad \frac{g^3-g^2}{1+g} = \frac{\sqrt{5}+1}{2}$.

$\frac{a}{x+}\,\frac{a^4}{x+}\,\frac{a^8}{x+}\,\frac{a^{12}}{x+\&c} = 1 - \frac{ax}{1+}\,\frac{a^2}{1-}\,\frac{a^2x}{1+}\,\frac{a^4}{1-\&c}$ nearly.

$\left(\frac{1}{64}\right)$. $\quad \sqrt{23} \quad 1-g^3 = g^2$
$\quad \sqrt{31} \quad 1-g^3 = g$

$(64.)$ $\quad \sqrt{11} \quad \frac{1}{2}-g^3 = g-g^2$
$\quad \sqrt{19} \quad \frac{1}{2}-g^3 = g^2$
$\quad \sqrt{27} \quad \frac{1}{2}-g^3 = g^2\sqrt[3]{3}$
$\quad \sqrt{43} \quad \frac{1}{2}-g^3 = g \quad \sqrt{67} \quad \frac{1}{2}-g^3 = g^2+g$.

$\sqrt{3}$ $\quad 2-\sqrt{3}$

$\sqrt{5}$ $\quad \left(\sqrt{\frac{3+\sqrt{5}}{4}} - \sqrt{\frac{\sqrt{5}-1}{4}}\right)^4$

$\sqrt{7}$ $\quad 8-3\sqrt{7}$

$\sqrt{9}$ $\quad \left(\sqrt{\frac{3+\sqrt{3}}{4}} - \sqrt{\frac{\sqrt{3}-1}{4}}\right)^8$

$\sqrt{13}$ $\quad \left(\sqrt{\frac{7+\sqrt{13}}{4}} - \sqrt{\frac{3+\sqrt{13}}{4}}\right)^4$

$\sqrt{15}$ $\quad (2-\sqrt{3})^2(4-\sqrt{15})$

$\sqrt{17}$ $\quad \left(\sqrt{\frac{3+\sqrt{4+\sqrt{17}}}{4}} - \sqrt{\frac{\sqrt{4+\sqrt{17}}-1}{4}}\right)^8$

$\sqrt{21}$

$\sqrt{25}$ $\quad \left(\sqrt{\frac{5+\sqrt{5}}{4}} - \sqrt{\frac{\sqrt{5}+1}{4}}\right)^8$

If n is a positive integer

$$\frac{1^{4n}}{(e^{\pi}-e^{-\pi})^2} + \frac{2^{4n}}{(e^{2\pi}-e^{-2\pi})^2} + \&c = \frac{n}{\pi}\left(\frac{B_{4n}}{8n} + \frac{1^{4n-1}}{e^{2\pi}-1} + \frac{2^{4n-1}}{e^{4\pi}-1} + \&c\right)$$

$$\frac{1}{2x^2} + \frac{1}{(x+1)^2} + \frac{1}{(x+2)^2} + \frac{1}{(x+3)^2} + \&c$$

$$= \frac{1}{2\pi x^3} + \frac{\pi}{3x} - \frac{\pi^2}{\sin^2\pi x\,(e^{2\pi x}-1)}$$

$$+ 4x\left\{\frac{1}{e^{2\pi}-1}\cdot\frac{1}{(1^2-x^2)^2} + \frac{2}{e^{4\pi}-1}\cdot\frac{1}{(2^2-x^2)^2} + \&c\right\}$$

$$+ 8\pi x^3\left\{\frac{1}{(e^{\pi}-e^{-\pi})^2}\cdot\frac{1}{1^4-x^4} + \frac{1}{(e^{2\pi}-e^{-2\pi})^2}\cdot\frac{1}{2^4-x^4} + \&c\right\}$$

If $\theta\frac{u}{\sqrt2} = v + \frac{1}{2}\cdot\frac{v^5}{5} + \frac{1.3}{2.4}\cdot\frac{v^9}{9} + \&c$, then

$$\frac{u^2}{2v^2} = \frac{1}{\sin^2\theta} - \frac{1}{\pi} - 8\left(\frac{\cos 2\theta}{e^{2\pi}-1} + \frac{2\cos 4\theta}{e^{4\pi}-1} + \&c\right).$$

$$\cot\theta + \frac{\theta}{\pi} + 4\left(\frac{\sin 2\theta}{e^{2\pi}-1} + \frac{\sin 4\theta}{e^{4\pi}-1} + \&c\right)$$

$$= \frac{u}{\sqrt2}\left\{\frac{1}{v} - \frac{1}{2}\cdot\frac{v^3}{3} - \frac{1.3}{2.4}\cdot\frac{v^7}{7} - \&c\right\}$$

$$\log\sin\theta + \frac{\theta^2}{2\pi} - 2\left\{\frac{\cos 2\theta}{1(e^{2\pi}-1)} + \frac{\cos 4\theta}{2(e^{4\pi}-1)} + \&c\right\}$$

$$= \log\frac{v\sqrt2}{u} + \frac{1}{3}\cdot\frac{v^4}{4} + \frac{1.5}{3.7}\cdot\frac{v^8}{8} + \frac{1.5.9}{3.7.11}\cdot\frac{v^{12}}{12} + \&c.$$

$$\frac{1}{2}\tan^{-1}v = \frac{\sin\theta}{\cosh\frac{\pi}{2}} + \frac{\sin 3\theta}{3\cosh\frac{3\pi}{2}} + \frac{\sin 5\theta}{5\cosh\frac{5\pi}{2}} + \&c$$

$$\frac{1}{4}\cos^{-1}v^2 = \frac{\cos\theta}{\cosh\frac{\pi}{2}} - \frac{\cos 3\theta}{3\cosh\frac{3\pi}{2}} + \frac{\cos 5\theta}{5\cosh\frac{5\pi}{2}} - \&c$$

$$\frac{\pi\theta}{8} - \frac{\sin\theta}{1^2\cosh\frac{\pi}{2}} + \frac{\sin 3\theta}{3^2\cosh\frac{3\pi}{2}} - \&c$$

$$= \frac{\sqrt2}{4u}\left\{\frac{v^3}{3} + \frac{2}{3}\cdot\frac{v^7}{7} + \frac{2.4}{3.5}\cdot\frac{v^{11}}{11} + \&c\right\},$$

If $\theta\frac{u}{2} = v - \frac{1}{2}\cdot\frac{v^5}{5} + \frac{1.3}{2.4}\cdot\frac{v^9}{9} - \&c$

$$2\tan^{-1}v = \theta + \frac{\sin 2\theta}{\cosh\pi} + \frac{\sin 4\theta}{2\cosh 2\pi} + \frac{\sin 6\theta}{3\cosh 3\pi} + \&c$$

$$\frac{\pi}{8} - \frac{1}{2}\tan^{-1}v^2 = \frac{\cos\theta}{\cosh\frac{\pi}{2}} - \frac{\cos 3\theta}{3\cosh\frac{3\pi}{2}} + \frac{\cos 5\theta}{5\cosh\frac{5\pi}{2}} - \&c$$

$$\frac{1}{2}\log\frac{1+v}{1-v} = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) + 4\left\{\frac{\sin\theta}{e^{\pi}-1} - \frac{\sin 3\theta}{3(e^{3\pi}-1)} + \&c\right\}$$

$$\frac{1}{2\pi x^4} + \coth\pi\left(\frac{1}{1+x^2+x^4} + \frac{1}{1-x^2+x^4}\right)$$

$$+ 2\coth 2\pi\left(\frac{1}{16+4x^2+x^4} + \frac{1}{16-4x^2+x^4}\right)$$

$$+ \quad \&c \qquad \&c$$

$$= \frac{\pi}{x^2\sqrt3}\cdot\frac{\sinh\pi x\sqrt3\,\sinh\pi x + \sin\pi x\sqrt3\,\sin\pi x}{(\cosh\pi x\sqrt3 - \cos\pi x)(\cosh\pi x - \cos\pi x\sqrt3)}$$

$$\Sigma\frac{1}{x} - \Sigma\frac{1}{x/3} + \frac{4}{x} - \log 3$$

$$= \frac{2}{3}\cdot\frac{1}{x^2} + \frac{2^3-2}{6} + \frac{4^3-4}{3x^2} + \frac{5^3-5}{6} + \frac{7^3-7}{5x^2} + \&c$$

$$\frac{1^{6n}}{(e^{\frac{\pi\sqrt3}{2}}+e^{-\frac{\pi\sqrt3}{2}})^2} - \frac{2^{6n}}{(e^{\pi\sqrt3}-e^{-\pi\sqrt3})^2} + \&c$$

$$+ \frac{n\sqrt3}{\pi}\left\{\frac{B_{6n}}{12n}\cos 3\pi n - \left(\frac{1^{6n-1}}{e^{\pi\sqrt3}+1} - \&c\right)\right\} = 0$$

n being a positive integer

$$(1+x)^n = 1 + nx(1+x)^{\frac{n-1}{2}} + \frac{n(n^2-1)}{4\,\underline{|3}}\, x^3 (1+x)^{\frac{n-3}{2}} +$$

$$+ \frac{n(n^2-1)(n^2-9)}{4^2\,\underline{|5}}\, x^5 (1+x)^{\frac{n-5}{2}} + \&c$$

$$\left(\frac{1+\sqrt{1+4x}}{2}\right)^n = 1 + nx(1+x)^{\frac{n-3}{2}} + \frac{n(n-5)(n-7)}{4\,\underline{|3}}\, x^3 (1+x)^{\frac{n-9}{2}}$$

$$+ \frac{n(n-7)(n-9)(n-11)(n-13)}{4^2\,\underline{|5}}\, x^5 (1+x)^{\frac{n-15}{2}}$$

$$\frac{1+(1+x)^n}{2} = (1+x)^{\frac{n}{2}} + \frac{n^2}{4\,\underline{|2}}\, x^2 (1+x)^{\frac{n-2}{2}}$$

$$+ \frac{n^2(n^2-2^2)}{4^2\,\underline{|4}}\, x^4 (1+x)^{\frac{n-4}{2}} + \&c$$

$$\frac{1}{2} + \frac{1}{2}\left(\frac{1+\sqrt{1+4x}}{2}\right)^n = (1+x)^{\frac{n}{2}} + \frac{n(n-4)}{4\,\underline{|2}}\, x^2 (1+x)^{\frac{n-6}{2}}$$

$$+ \frac{n(n-6)(n-8)(n-10)}{4^2\,\underline{|4}}\, x^4 (1+x)^{\frac{n-12}{2}} + \&c.$$

IV

2	2	144	110880
3	4	160	166320
4	6	168	221760
6	12	180	277200
8	24	192	332640.
9	36	200	498960
10	48	216	554400
12	60	224	665280
16	120	240	720720
18	180	256	1081080
20	240	288	1441440
24	360	320	2162160
30	720	336	2882880
32	840	360	3603600
36	1260	384	4324320
40	1680	400	6486480
48	2520	432	7207200
60	5040	448	8648640
64	7560	480	10810800
72	10080	504	14414400
80	15120	512	17297280
84	20160	576	21621600
90	25200	600	32432400
96	27720	640	36756720
100	45360	672	43243200
108	50400	720	61261200
120	55440	768	73513440
128	83160	800	110270160

Search: **seq:2,4,6,12,24,36**

Displaying 1-10 of 33 results found. page 1 2 3 4

Sort: relevance | references | number | modified | created Format: long | short | data

A002182 Highly composite numbers: numbers n where d(n), the number of divisors of n (A000005), increases to a record. +30
403
(Formerly M1025 N0385)

1, **2**, **4**, **6**, **12**, **24**, **36**, 48, 60, 120, 180, 240, 360, 720, 840, 1260, 1680, 2520, 5040, 7560, 10080, 15120, 20160, 25200, 27720, 45360, 50400, 55440, 83160, 110880, 166320, 221760, 277200, 332640, 498960, 554400, 665280, 720720, 1081080, 1441440, 2162160

(list; graph; refs; listen; history; text; internal format)

OFFSET 1,2

COMMENTS Where record values of d(n) occur: d(n) > d(k) for all k < n.

A002183 is the RECORDS transform of A000005, i.e., lists the corresponding values d(n) for n in A002182.

Flammenkamp's page also has a copy of the missing Siano paper.

Highly composite numbers are the product of primorials, A002110. See A112779 for the number of primorial terms in the product of a highly composite number. - Jud McCranie, Jun 12 2005

Sigma and tau for highly composite numbers through the 146th entry conform to a power fit as follows: log(sigma)=A*log(tau)^B where (A,B) =~ (1.45,1.38). - Bill McEachen, May 24 2006

a(n) often corresponds to P(n,m) = number of permutations of n things taken m at a time. Specifically, if start=1, pointers 1-6, 9, 10, 13-15, 17-19, 22, 23, 28, 34, 37, 43, 52, ... An example is a(37)=665280, which is P(12,6)=12!/(12-6)!. - Bill McEachen, Feb 09 2009

Concerning the previous comment, if m=1, then P(n,m) can represent any number. So let's assume m > 1. Searching the first 1000 terms, the only indices of terms of the form P(n,m) are 4, 5, 6, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 27, 28, 31, 34, 37, 41, 43, 44, 47, 50, 52, and 54. Note that a(44) = 4324320 = P(2079,2). See A163264. - T. D. Noe, Jun 10 2009

A large number of highly composite numbers have 9 as their digit root. - Parthasarathy Nambi, Jun 07 2009

Because 9 divides all highly composite numbers greater than 1680, those numbers have digital root 9. - T. D. Noe, Jul 24 2009

See A181309 for highly composite numbers that are not highly abundant.

a(n) is also defined by the recurrence: a(1) = 1, a(n+1)/sigma(a(n+1)) < a(n) / sigma(a(n)). - Michel Lagneau, Jan 02 2012 [NOTE: This "definition" is wrong (a(20)=7560 does not satisfy this inequality) and incomplete: It does not determine a sequence uniquely, e.g., any subsequence would satisfy the same relation. The intended meaning is probably the definition of the (different) sequence A004394. - M. F. Hasler, Sep 13 2012]

Up to a(1000), the terms beyond a(5) = 12 resp. beyond a(9) = 60 are a multiples of these. Is this true for all subsequent terms? - M. F. Hasler, Sep 13 2012 [Yes: see EXAMPLE in A199337! - M. F. Hasler, Jan 03 2020]

864	122522400
896	14.7026880
960	183783600
1008	245044800
1024	294053760
1152	367567200
1200	551350800
1280	698377680
1344	735134400
1440	1102701600
1536	1396755360
1600	2095133040
1680	2205403200
1728	2327925600
1792	2793510720
1920	3491888400
2016	4655851200
2048	5587021440
2304	6983776800
2400	10475665200
2688	13967553600
2880	20951330400
3072	27935107200
3360	4190266[illegible]
3456	488864[illegible]1600
3584	64250746560
3600	733[illegible]96[illegible]6400
3840	8031[illegible]433200
4032	97772875200
4096	128501493120
4320	146659312800.

$B_0 = -1$; $B_2 = \frac{1}{6}$; $B_4 = \frac{1}{30}$; $B_6 = \frac{1}{42}$; $B_8 = \frac{1}{30}$; $B_{10} = \frac{5}{66}$; $B_{12} = \frac{691}{2730}$; $B_{14} = \frac{7}{6}$; $B_{16} = \frac{3617}{510}$; $B_{18} = \frac{43867}{798}$; $B_{20} = \frac{174611}{330}$; $B_{22} = \frac{854513}{138}$; $B_{24} = \frac{236364091}{2730}$;

$B_{26} = \frac{8553103}{6}$; $B_{28} = \frac{23749461029}{870}$; $B_{30} = \frac{8615841276005}{14322}$; $B_{32} = \frac{7709321041217}{510}$;

$$B_{34} = \frac{2577687858367}{6};$$

$$B_{36} = \frac{26315271553053477373}{1919190}$$

$$B_{38} = \frac{2929993913841559}{6}; \text{ \&c } B_\infty = \infty$$

19. If n be an even integer,

i. B_n is a fraction & $2(2^n-1)B_n$ is an integer

ii. The numerator of B_n in its lowest terms is divisible by the greatest odd measure of n prime to the denominator and the quotient is a prime number

iii. The denominator of B_n is the continued product of prime numbers next to the factors of n including unity and the number itself.

$\sqrt{2} = 1.4142135623730950488017$

~~$\sqrt{3} = 1.7320$~~

$3 — 64x^{24}$

$7 — \frac{x^{24}}{64}$

$3 — x^3 = \frac{4}{2}$

$11 — x^3 - x^2 + x = \frac{4}{2}$

$19 — x^3 + x^2 = \frac{4}{2}$

$27 — x^3 + x^2 \sqrt[3]{3} = \frac{4}{2}$

$35 —$

$43 — x^3 + x = \frac{4}{2}$

$51 —$

$59 —$

$67 — x^3 + x^2 + x = \frac{4}{2}$

$75 —$

$7 — x^3 = 1$

$15 —$

$23 — x^3 + x^2 = 1$

$31 — x^3 + x = 1$

$39 —$

47

55

63